9-4 Inverse Functions

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x).

Notation:

$$f^{-1}(x)$$

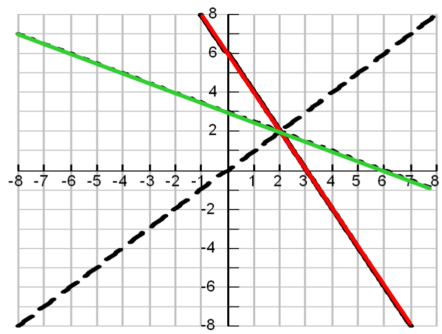
Represents the inverse of the function $f(x)$

Horizontal-Line Test

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

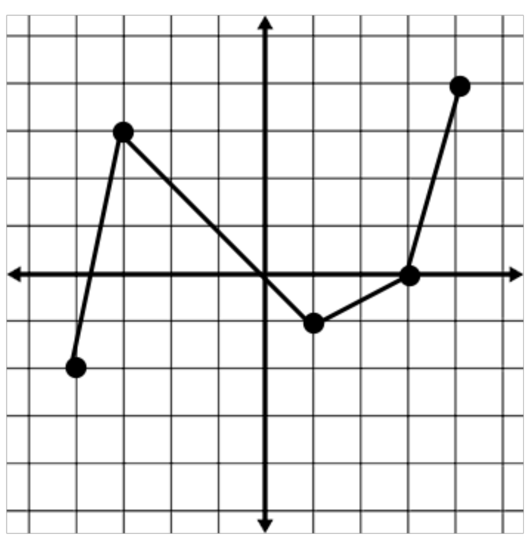
If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.

Show $f(x) = 6 - 2x$ and $g(x) = \dfrac{6 - x}{2}$ are inverses graphically.



f(x):	(1,4)	(3,0)	(4,-2)
g(x):	(4,1)	(0,3)	(-2,4)

Graph the inverse of the graph. (Use y=x to find inverse points)



To find the inverse equation of a function

1. Change $f(x)$ to y.
2. Interchange x and y
3. Solve for y
4. Change new y to $f^{-1}(x)$

Find the inverse of each function. List any domain restrictions if applicable.

$$f(x) = x^2 + 1$$

$$g(x) = \frac{x+1}{2x+3}$$

Find the inverse of each function.

$$h(x) = 2x^3 + 3 \qquad g(x) = \sqrt[3]{x} - 3$$

We can verify that two functions are inverses of each other by determining if the composition of the two functions are both equal to x.

$$f \circ g = x \qquad g \circ f = x$$

$$f \circ f^{-1} = x \qquad f^{-1} \circ f = x$$

Use composition to determine if the following functions are inverses of each other.

$$f(x) = 5x + 1$$

$$g(x) = \frac{x-1}{5}$$