7.1 Zeros of a Polynomial

Divide the following polynomials

$$x+4\overline{)3x^2+7x-20}$$

$$\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$$

Identify the zeros of the following and explain what that means graphically.

$$f(x)=(x+2)(x-1)(x+3)$$

Write the function in standard form and state the relationship between the degree and zeros of the function

Remainder Theorem:

For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

Factor Theorem:

If the remainder in $p(x) = (x - a)q(x) + p(a)$ is 0, then $p(x) = (x - a)q(x)$, which tells you that $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then you can write $p(x)$ as $p(x) = (x - a)q(x)$, and when you divide $p(x)$ by $(x - a)$, you get the quotient $q(x)$ with a remainder of 0.

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(B) $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5; (x + 1)$

So, $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5 =$ __________.

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Example 3 Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.

(A) $p(x) = x^3 + 3x^2 - 4x - 12; (x + 3)$

Your Turn

Determine whether the given binomial is a factor of the polynomial $p(x)$. If it is, find the remaining factors of $p(x)$.

8. $p(x) = 2x^4 + 8x^3 + 2x + 8; (x + 4)$

9. $p(x) = 3x^3 - 2x + 5; (x - 1)$

Rational Root Theorem:

If all coefficients are integers and the constant is not 0, then all possible rational roots are:

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^3 + 2x^2 - 19x - 20$$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$$

Find all the zeros $f(x) = x^3 - 2x^2 - 8x$

Find all the zeros of: $2x^4 - 7x^3 - 8x^2 + 14x + 8$

Find all the zeros of: $f(x) = x^3 + x^2 - 14x + 6$

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Find the polynomial function with a leading coefficient of 2 that has the given degree and zeros: degree 3, zeros -2, 4, 1