5-1 Pythagorean Identities Identity: equality that is true for all values of the domain for both expressions as long as they are both defined

$$
\tan \theta \cdot \cos \theta=\sin \theta
$$

this is true for $\operatorname{all} \theta$, as long as $\sin \theta, \cos \theta$, and $\tan \theta$ are defined

## Reciprocal \& Quotient Relationships

$$
\sin \theta=\frac{1}{\csc \theta} \quad \csc \theta=\frac{1}{\sin \theta} \quad \tan \theta=\frac{\sin \theta}{\cos \theta}
$$

$$
\cos \theta=\frac{1}{\sec \theta} \quad \sec \theta=\frac{1}{\cos \theta} \quad \cot \theta=\frac{\cos \theta}{\sin \theta}
$$

$$
\tan \theta=\frac{1}{\cot \theta} \quad \cot \theta=\frac{1}{\tan \theta}
$$

Odd/Even Identities


$$
\begin{aligned}
& \cos (-x)=\cos x \\
& \sec (-x)=\sec x
\end{aligned}
$$



$$
\begin{aligned}
& \sin (-x)=-\sin x \\
& \csc (-x)=-\csc x
\end{aligned}
$$



$$
\tan (-x)=-\tan x
$$

$$
\cot (-x)=-\cot x
$$



Pythagorean Relationships

$$
\begin{aligned}
\sin ^{2} \theta+\cos ^{2} \theta & =1 \\
\sin ^{2} \theta & =1-\cos ^{2} \theta \\
\cos ^{2} \theta & =1-\sin ^{2} \theta
\end{aligned}
$$

Now lets divide by $\cos ^{2} \theta$

$$
\sin ^{2} \theta+\cos ^{2} \theta=1
$$

Pythagorean Relationships

$$
\begin{aligned}
& 1+\tan ^{2} \theta=\sec ^{2} \theta \\
& 1=\sec ^{2} \theta-\tan ^{2} \theta \\
& \tan ^{2} \theta=\sec ^{2} \theta-1
\end{aligned}
$$

Now lets divide by $\sin ^{2} \theta$

$$
\sin ^{2} \theta+\cos ^{2} \theta=1
$$

$1+\cot ^{2} \theta=\csc ^{2} \theta$
$1=\csc ^{2} \theta-\cot ^{2} \theta$
$\cot ^{2} \theta=\csc ^{2} \theta-1$

## Pythagorean Relationships

$$
\sin ^{2} \theta+\cos ^{2} \theta=1
$$

$$
1+\tan ^{2} \theta=\sec ^{2} \theta
$$

$$
1+\cot ^{2} \theta=\csc ^{2} \theta
$$

Simplify:
$\cot x \tan x \quad \sin \theta \csc \theta$

$$
\sin x \csc (-x) \quad \frac{\sec ^{2} x}{\tan ^{2} x}
$$

Simplify
$\sec x-\sin x$
$\sin x \cos x$

Establish the Identity:
$\csc x \cos x=\cot x$
$\left(1-\sin ^{2} x\right)\left(1+\tan ^{2} x\right)=1$
Simplify:

$$
\frac{1}{\sin \alpha-1}-\frac{1}{\sin \alpha+1}
$$

$$
\cos x(\tan x+\cot x)=\csc x
$$

