3-1

3-1 Defining and evaluating logarithms

Explain 1 Converting Between Exponential and Logarithmic Forms of Equations

In general, the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, has the logarithmic function $f^{-1}(x) = \log_b x$ as its inverse. For instance, if $f(x) = 3^x$, then $f^{-1}(x) = \log_3 x$, and if $f(x) = \left(\frac{1}{4}\right)^x$, then $f^{-1}(x) = \log_{\frac{1}{4}} x$. The inverse relationship between exponential functions and logarithmic functions also means that you can write any exponential equation as a logarithmic equation and any logarithmic equation as an exponential equation.

Exponential Equation	Logarithmic Equation
$b^x = a$	$\log_b a = x$

$$b > 0, b \neq 1$$

Switch between Log and exponential forms

Exponential Equation	Logarithmic Equation
$3^5 = 243$	
	$\log_4 \frac{1}{64} = -3$
$\left(\frac{3}{4}\right)^r = s$	
	$\log_{\frac{1}{5}} v = w$

The natural logarithm:

$y = \ln x$ is equivalent to $x = e^y$

The common logarithm:

$y = \log x$ is equivalent to $x = 10^y$

Exponential Equation	Logarithmic Equation
$e^5 \approx 148.4$	
	$\ln 6 \approx 1.8$
$10^5 = 100{,}000$	
	$\log 1{,}000 = 3$

P 793

If $f(x) = \log_{\frac{1}{2}} x$, find $f(4)$, $f\left(\frac{1}{32}\right)$ and $f\left(2\sqrt{2}\right)$.

$f(4) = x$

$\log_{\frac{1}{2}} 4 = x$

So, $f(4) = \square$.

$f\left(\frac{1}{32}\right) = x$

$\log_{\frac{1}{2}} \frac{1}{32} = x$

So, $f\left(\frac{1}{32}\right) = \square$.

$f\left(2\sqrt{2}\right) = x$

$\log_{\frac{1}{2}} 2\sqrt{2} = x$

$x = \square$

So $f\left(2\sqrt{2}\right) = \square$.

Find the exact value without a calculator

$\log_2 32$ $\quad$ $\log_4 \frac{1}{16}$

$\log 10000000$ $\quad$ $\log .00001$

You try

$\log_5 25$ $\quad$ $\log_2 \frac{1}{8}$

$\log 1000$ $\quad$ $\log .001$

Your Turn

Use a scientific calculator to find the common logarithm and the natural logarithm of the given number. Verify each result by evaluating the appropriate exponential expression.

11. 0.25

12. 4

The acidity level, or pH, of a liquid is given by the formula $\text{pH} = \log \frac{1}{[H^+]}$ where $[H^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. In a typical chlorinated swimming pool, the concentration of hydrogen ions ranges from 1.58×10^{-8} moles per liter to 6.31×10^{-8} moles per liter. What is the range of the pH for a typical swimming pool?